

Isospin effects in the thermodynamics of finite nuclei

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Abstract

It has been proposed that multifragmentation can be related to the liquid-gas phase transition of nuclear matter. We study the statistical properties of finite nuclear matter near the phase transition with the help of a Lattice Gas Model (LGM). The original version of LGM with only one type of charge-neutral particles is well known to feature the properties of the liquid-gas phase transition. In this contribution, we address the effect of Coulomb and isospin dependence interaction for the finite nuclei transition, and study the symmetry energy properties of finite temperature systems.

In the last decade, there has been a growing interest for the measurement of the symmetry coefficient c_{sym} of the nuclear equation of state, through experiments with heavy ion collisions.

At zero temperature, most mean-field-based nuclear equations of state can be approximately reproduced through the density (ρ) functional form [1] :

$$c_{sym}(\rho) = c_{sym}^0 \left(\frac{\rho}{\rho_0} \right)^\gamma \quad (1)$$

where ρ_0 is the saturation density, and γ is a coefficient which determines the stiffness of the symmetry coefficient. The values of γ can widely differ among different EOS parametrisations ($0.5 \div 2$) and are not presently constraint by experimental measurements.

One proposed technique is the so-called isoscaling analysis. It is well known [2] that the ratio $R_{21}(N, Z)$ of isotope yields $Y_i(N, Z)$ measured in two reactions similar in temperature but different in isospin labelled (1,2) has an exponential behavior according to :

$$R_{21}(N, Z) = \frac{Y_2(N, Z)}{Y_1(N, Z)} \propto \exp(\alpha N + \beta Z) \quad (2)$$

where α and β are the isoscaling parameters.

Different data have been published showing that α decreases with increasing the violence of the collision [2, 3, 4]. Since an increasing collision violence is associated to increasing temperature and/or decreasing density at the time of fragment formation, this suggests that we may access the density and/or temperature evolution of the symmetry term. Indeed at thermal equilibration and in the framework of the grand canonical ensemble, α and c_{sym} are approximately related by [5] :

$$\alpha T \approx 4c_{sym} \left(\left(\frac{Z_1}{A_1} \right)^2 - \left(\frac{Z_2}{A_2} \right)^2 \right) \quad (3)$$

where T is the common temperature of the two systems, and Z_i, A_i are the corresponding charges and masses.

Heavy ion collisions are probably the best probe of nuclear properties in excitation energy and density conditions far from the ground state. However, the connexion between the symmetry energy coefficient appearing in eqs.1 and 3 is far from being clear. Indeed the fragmented configurations accessed by isoscaling analysis are dishomogeneous and present high order correlations; it is therefore not clear whether, even at thermal equilibrium, the associated energy functional only depends on the global density and temperature as obtained in a mean-field based picture. Moreover eq.3 has been derived in the framework of macroscopic statistical models [5] where many body correlations are supposed to be entirely exhausted by clusterisation and clusters are described as independent degrees of freedom. If the validity of isoscaling eq.2 in microscopic theories is well settled [6], the connexion of α and c_{sym} eq.3, in such models has never been proved.

To progress on these issues, it is interesting to consider a microscopic model simple enough to be exactly solvable through Monte-Carlo simulations without any mean-field or independent cluster approximation.

In this contribution, we propose to study the temperature and/or density dependence of the symmetry coefficient c_{sym} in a lattice gas model.

1 Description of the model

We use a cubic lattice of linear site $L = 20$, where each cell i is characterised by four degrees of freedom : one discrete variable σ_i for isospin ($\sigma_p = 1, \sigma_n = -1, \sigma_0 = 0$) and three continuous variables \vec{p}_i for the momentum.

The hamiltonian of the system follows:

$$H = \sum_{\langle i,j \rangle} \epsilon_{\sigma_i \sigma_j} \sigma_i \sigma_j + \sum_{\sigma_i = \sigma_j = 1, (i \neq j)} \frac{I_c}{r_{ij}} + \sum_{i=1}^{L^3} \frac{p_i^2}{2m} \sigma_i^2 \quad (4)$$

where $\langle i, j \rangle$ are nearest neighbour cells, $\epsilon_{\sigma_i \sigma_j}$ is the coupling between nearest neighbour ($\epsilon_1 = 0, \epsilon_{-1} = 5.5 MeV$), $I_c (= 1.44 MeV/fm)$ is the Coulomb interaction between all protons in the lattice, and r_{ij} is the distance between sites i and j . The last part of the interaction is the kinetic term where $m = 939 MeV$ is the nucleon mass.

Calculations are made in the isobar canonical ensemble, which has been shown to be the correct canonical ensemble to describe unbound systems in the vacuum [7].

The partition function is then:

$$Z = \sum_{(n)} \exp \left(-\beta \left(H^{(n)} + P R^{3(n)} \right) \right) \quad (5)$$

where the sum runs over all the possible realisations of the system, and $R^{3(n)}$ is the global extension of the system for each partition (n) define as:

$$R^{3(n)} = \frac{2 \left(\sum r_i^3 \sigma_i^2 \right)^{(n)}}{\left(\sum \sigma_i^2 \right)^{(n)}} \quad (6)$$

The statistical average at a given value of T and P ($\langle R^{3(n)} \rangle$) will be noted R^3 in the following.

2 Phase transition

In its original version with only one type of particles and closest neighbours interaction, the Lattice Gas Model (LGM) is well known to exhibit a first order transition line at low temperature ending to a second order critical point [8, 9]. In this case, with a short range isovector coupling and a long range repulsive interaction, the phase diagram is not qualitatively modified [10]. Indeed we find a phase coexistence line ended by the critical point,

characterized by its temperature T_c and pressure P_c . In the following, all calculations are made at the subcritical pressure $P = 5. \times 10^{-5} MeV.fm^{-3}$.

In table 1, we compile the transition temperature T_t for different systems at the chosen pressure. The inclusion of Coulomb and isospin dependence results in a lower transition temperature than for a scalar interaction without coulomb. With the realistic hamiltonian, in all cases the isospin dependence is weak on the transition temperature, less than few percents.

System	(75, 75) _{scalar}	(75, 75)	(85, 65)	(88, 62)	(91, 59)
$T_t(MeV)$	3.17	2.27	2.285	2.290	2.295

Table 1: Liquid-Gas transition temperature of different systems of $A = 150$ nucleons at subcritical pressure $P = 5.10^{-5} MeV.fm^{-3}$.

To explore the finite temperature symmetry energy of the model, we will now use a macroscopic parametrisation in order to connect the microscopic and macroscopic properties of these systems.

3 From microscopic to macroscopic

As shown in the previous section, finite temperature Lattice Gas systems are strongly dishomogeneous and clusterized because of the presence of the phase transition. In this situation, it is not clear whether the global energetics of the system, including its symmetry properties, can be described by a macroscopic parametrisation depending on the average density as in the nuclear mean-field.

To explore this issue and in order to extract the symmetry energy of our systems, we use a liquid-drop (macroscopic) parametrisation for the interaction energy of the system, which reads:

$$E_{int}^{LD}(\delta, \rho, T) = a_v(\rho, T)A + a_s(\rho, T)A^{2/3} + c_{sym}(\rho, T)A\delta^2 + \alpha_c(\rho, T)\frac{Z^2}{R} \quad (7)$$

Here δ is the isospin asymmetry $(N - Z)/(N + Z)$, T is the temperature and $\rho = A/(4/3\pi R^3)$ is an estimation of the average density of the system, where the mean cubic radius from eq.6 is calculated, excluding the monomers ($A = 1$).

This parametrisation uses four macroscopic parameters: a_v is associated to the volume energy, a_s corresponds to the surface energy, c_{sym} is the coefficient related to the symmetry energy, α_c corresponds to the Coulomb contribution of this interaction.

3.1 Temperature dependence

In order to extract all these parameters, we fit our systems at different temperatures. The results obtained from the parametrisation are plotted versus the true values coming out from the simulation on the figure 1-*left*. We observe a good agreement between the two results (close to the line $E_{int}^{LD} = E_{int}$). The values of the macroscopic coefficients are plotted as a function of the temperature on the figure 1-*right*. α_c is constant with temperature, while a_v and a_s decrease with increasing temperature, and c_{sym} decreases with increasing temperature with a bump around the transition temperature for the considered pressure ($T_t \approx 2.25 MeV$).

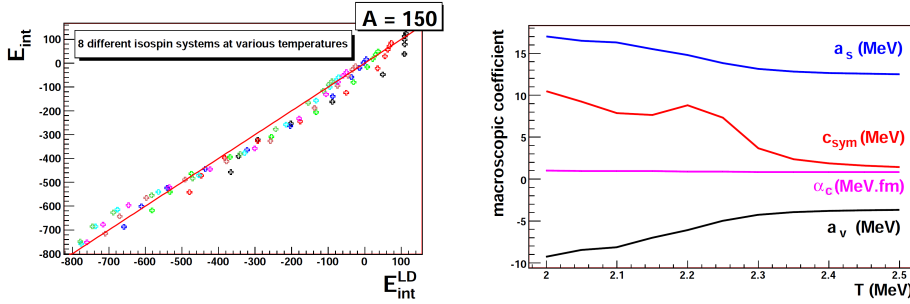


Figure 1: Left: Correlation between the liquid drop parametrisation (eq.7) and the average LGM energy for eight different systems at different temperatures. Right: Evolution of the macroscopic coefficients with the temperature.

A very strong temperature dependence is observed, at variance with nuclear mean-field calculations [1]. However, up to now, we have extracted the temperature dependence of the macroscopic coefficient without looking at the density dependence, while a temperature variation induces a change in the average system size for these constant pressure calculations [11].

3.2 Density dependence

On figure 2-*left*, we plot the evolution of the density with the temperature. We observe a decrease of the density with increasing temperature similar to the behavior of a_v and a_s .

The temperature variation of a_v and a_s can be easily understood as a simple effect of this density change, as we now explain.

To avoid the interference of the coulomb and symmetry effects, we consider an isospin symmetric system ($N = 75$, $Z = 75$) and switch off the

coulomb interaction.

To distinguish the respective role of ρ and T , we sort out the temperature dependence induced by the density variation $\rho(T)$ as:

$$a_{v,s}(\rho, T) \propto f_{v,s}(\rho(T)) a'_{v,s}(\rho, T) \quad (8)$$

If the temperature dependence shown in figure 1 is only due to the change with temperature of the average size as the mean-field approximation, eq.8 should be fulfilled with a'_v and a'_s as constants. This is indeed what is observed once the total bulk energy is plotted as a function of the mean radius, as shown in figure 2-*right*. The observed functional dependence of $f(\rho)$ can be easily understood from simple geometrical considerations :

$$f_v(\rho(T)) \propto \frac{\rho(T)}{\rho_0} \propto R^{-3}(T) \quad f_s(\rho(T)) \propto \frac{S(T)}{V(T)} \propto R^{-1}(T) \quad (9)$$

where S and V represent the average system surface and volume.

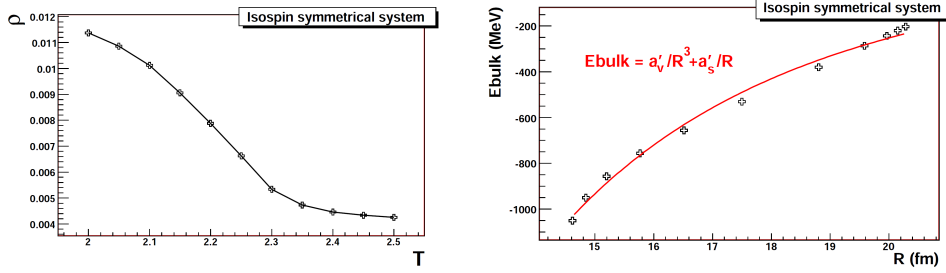


Figure 2: Left: density evolution in fonction of the temperature. Right: Total bulk energy in fonction of the radius of the system.

A similar analysis for the symmetry coefficient with the realistic hamiltonian including Coulomb is currently under progress.

Conclusions

In order to connect the microscopic properties of the nuclear equation of state (symmetry energy and its dependence on temperature and/or density) and experimental fragment observables like isoscaling, we have used a lattice gas model with an isospin dependent short range and a long range coulomb interaction. In this case, we observe a liquid-gas transition at lower temperature than the liquid gas transition with a purely nuclear isospin independent interaction. The coupling of an isovector and coulomb interaction induces

another first order transition, a kind of fission-fusion transition at lower temperature than the liquid-gas one. We have also established a macroscopic parametrisation of the model, with four parameters which are function of temperature and density (a_v , a_s , c_{sym} , α_c). For isospin symmetric systems, and as long as the coulomb interaction is ignored, the whole temperature dependence appears to be exhausted by the total average density dependence, even in the phase transition region where the system is highly dishomogeneous and clusterised. This finding suggests that it may be possible to access the density dependence of c_{sym} in multifragmentation experiments, as it is actively searched for in several recent experimental analysis [3, 4, 5]. The connection between isoscaling observables and the symmetry energy for finite temperature within this model is currently in progress [10].

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